~~Open~~ Closed Sphere

$$\{x \in X ; d(x,a) \leq r\}$$

$$\{x \in X : d(x,a) < r\}$$

$$\downarrow$$

Open Sphere

Open Sphere in R.

The usual metric for R is defined by $d(x,y) = |x-y| \ \forall \ x,y \in R$.

If $a \in R$, the open sphere withe centre 'a' and radius 'r' is given by

$$S(a,r) = \{x \in R : d(x,a) < r\}$$

$$S(a,r) =]a-r, a+r[$$

Open Sphere in N.

The usual metric for N is defined by $d(x,y) = |x-y| \ \forall \ x,y \in N$

let $n_0 \in N$ & $r > 0$

then $S(n_0, r) =$ the set of all natural nos. which lie b/w $n_0 - r$ and $n_0 + r$.

~~Since~~ i.e., $S(n_0, r) = \{n_0 \in N : d(x, n_0) < r\}$

$$=]n_0 - r, n_0 + r[$$

Open Sphere in R^2

The usual metric in R^2 is defined as. $d(x,y) = \sqrt{(x_1-y_1)^2+(x_2-y_2)^2}$

where $x=(x_1,x_2)$ and $y=(y_1,y_2)$

Hence, the open sphere with centre 'a' and radius 'r' is given by

$S(a,r) = \{(x,a) \in R^2 : d(x,a) < r\}$

$= \sqrt{(x_1-a_1)^2+(x_2-a_2)^2} < r$

$\Rightarrow S(a,r) = \{(x,y) : (x-a_1)^2+(y-a_2)^2 < r^2\}$

Examples

1) Open Sphere in R.

If $S(4,2) =]4-2, 4+2[=]2,6[$

2) Open Sphere in N

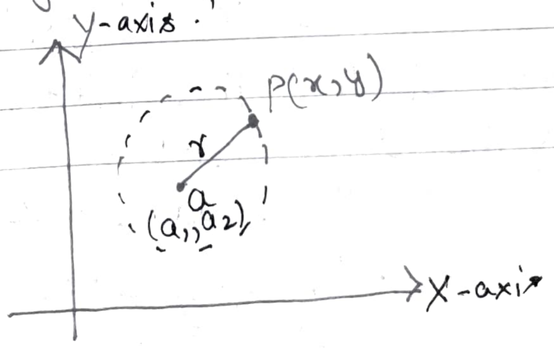
If $S(5,3) = \{n : n \in N \ \& \ 5-3<n<5+3\}$

$= \{n : n \in N \ \& \ 2<n<8\}$

$= \{3,4,5,6,7\}$.

3) Open sphere in R^2

If the centre is 'a', radius $= r$, and point $P(x,y)$ then the open spere in R^2 is given by all points that lie within the circle excluding circumfrence as shown below:



Ques Describe open spheres of unit radius $(0,0)$ for each of the following metrices of R^2.

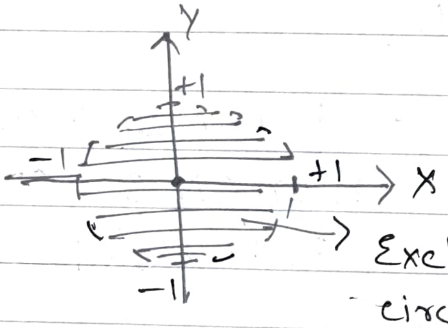
(i) $d_1(x,y) = \sqrt{(x_1-x_2)^2+(y_1-y_2)^2}$

(ii) $d_2(x,y) = |x_1-x_2| + |y_1-y_2|$.

Soln (i) The open sphere with centre $a=(0,0)$ & radius r is given by $S(a,r) = \sqrt{(x-0)^2+(y-0)^2} < r$

$\Rightarrow x^2+y^2<1$ $(\because r=1$

$\therefore$ the open sphere consists of all points of ~~the same~~ the plane which lie b/w the circle $x^2+y^2=r^2$.



Excluding the circumfrence.

(ii) $a=(0,0)$, $r=1$

$S(a,1) = \{x,y \in R^2 : |x-0|+|y-0|<1\}$

$= |x|+|y|<1$

We get that the interior is bounded by the lines $x+y=1$, $x-y=1$, $-x-y=1$ and $-x+y=1$